FULLY WORKED SOLUTIONS

Chapter 9: Bats, balls and boots — collisions in sport

Chapter questions

1. $F\Delta t = m\Delta v$

   $F \times 0.35 = 70 \times (0 - 10)$

   $F = -700/0.35 = -2000$ N

2. $F\Delta t = m(v - u)$

   $700 \times 0.4 = 60 \times (v - 0)$

   $v = 4.7$ m s^{-1}

3. (a) $\Delta p = F\Delta t = 1060 \times 0.5 = 530$ Ns

   (b) $v = \Delta p/m = 530/60 = 8.8$ m s^{-1}

   (c) $u_y = 8.33$ m s^{-1}

   $v_y = 0$

   $v_y^2 = u_y^2 + 2ay$

   $0^2 = (8.83)^2 + 2 \times (-9.8) \times y$

   $0 = 78 - 19.6y$

   $19.6y = 78$

   $y = 3.98 \text{ m} \approx 4.0 \text{ m}$

4. $m_w u_w + m_y u_y = m_w v_w + m_y v_y$

   $(m \times 3) + (m \times 0) = (m \times 0.5) + m v_y$

   $3 = 0.5 + v_y$

   $v_y = 2.5$ m s^{-1}

5. $m_b u_b + m_j u_j = m_b v_b + m_j v_j$

   $1.6u_b + (0.285 \times 0) = (1.6 \times 0.5) + (0.285 \times 3)$

   $1.6u_b = 1.655$

   $u_b = 1.0$ m s^{-1}

6. $m_b u_b + m_p u_p = m_b v_b + m_p v_p$

   $(7.2 \times 10) + (1.6 \times 0) = 7.2v_b + (1.6 \times 10)$

   $72 = 7.2v_b + 16$

   $56 = 7.2v_b$

   $v_b = 7.8$ m s^{-1}

7. $0 = m_b v_b + m_r v_r$

   $0 = (0.0036 v_b) + (4.6 \times 0.6)$

$-2.76 = 0.0036v_b$

$v_b = -767 \text{ m s}^{-1} \approx -770 \text{ m s}^{-1}$

8. $0 = m_c v_c + m_b v_b$

$0 = 1200v_c + (5 \times 120)$

$-600 = 1200v_c$

$v_c = -0.5 \text{ m s}^{-1}$

9. Resolving momenta in the x-direction:

$m_5u_5 + m_4u_4 = m_5v_5 + m_4v_4$

$(5 \times 4) + (4 \times (-3)) = (5 \times 0) + 4v_4$

$20 - 12 = 4v_4$

$v_4 = 2 \text{ m s}^{-1}$

Resolving momenta in the y direction:

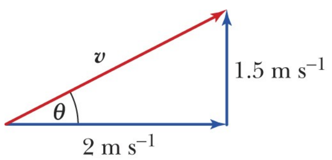
$m_5u_5 + m_4u_4 = m_5v_5 + m_4v_4$

$0 + 0 = (5 \times (-1.2)) + 4v_4$

$6 = 4v_4$

$v_4 = 1.5 \text{ m s}^{-1}$

Adding the x and y components of the ball's velocity:



$v^2 = 2^2 + 1.5^2$

(a) $v = 2.5 \text{ m s}^{-1}$

(b) $\theta = \tan^{-1}(1.5/2) = 36.9°$ north of east

10. $E_k = ½mv^2 = ½ \times 0.16 \times (15)^2 = 18 \text{ J}$

11. $v^2 = 2E_k/m = 2 \times 22/0.42 = 104.8$

$v = 10.2 \text{ m s}^{-1}$

12. Bowling ball: $E_k = ½ \times 6 \times 5^2 = 75 \text{ J}$

Cricket ball: $E_k = ½ \times 0.16 \times (15)^2 = 18 \text{ J}$

The bowling ball has the greatest kinetic energy

Review questions

6. (a) $p = mv = 50 \times 5 = 250 \text{ Ns}$

(b) $p = mv = 0.004 \times 320 = 1.28 \text{ Ns}$

(c) $p = mv = 0.420 \times 20 = 8.4$ Ns

7. (a) $\Delta p = F\Delta t = 18 \times 0.15 = 2.7$ Ns

(b) $a = \dfrac{F}{m} = \dfrac{18}{0.16} = 112.5$ m s^{-2}

8. (a) $\Delta p = m\Delta v = 75\ (0 - 3.4) = -255$ kg m s^{-1}

(b) $F = \dfrac{\Delta p}{\Delta t} = \dfrac{-255}{0.10} = -2\ 550$ N

9. (a) $\Delta \mathrm{p} = m\Delta v = 90\ (0 - 7) = -630$ kg m s^{-1}

(b) $F = \dfrac{\Delta p}{\Delta t} = \dfrac{630}{0.08} = 7\ 875$ N

10. $u_s = 0$, $u_A = 2$ m s^{-1}, $m_A = 60$ kg, $m_s = 70$ kg, $v_A = 0$

(a) $m_A u_A + m_s u_s = m_A v_A + m_s v_s$

$(60 \times 2) + (70 \times 0) = (60 \times 0) + (70 \times \mathrm{v}_s)$

$120 = 70v_s$

$v_s = 1.7$ m s^{-1}

(b) $\Delta p_A = m\Delta v_A = 60 \times (0 - 2) = -120$ Ns

(c) 120 Ns

(d) 120 Ns

(e) $m_A u_A + m_S u_S = m_{A+S} v_{A+S}$

$(60 \times 2) + (70 \times 0) = (60 + 70)v_{A+S}$

$120 = 130v_{A+S}$

$v_{A+S} = 0.9$ m s^{-1}

11. $m_t = 3$ kg, $m_A = 0.045$ kg, $u_A = 20$ m s^{-1}, $u_t = 0$, $v_A = 12$ m s^{-1}

$m_t u_t + m_A u_A = m_t v_t + m_A v_A$

$(3 \times 0) + (0.045 \times 20) = 3v_t + (0.045 \times 12)$

$0.9 = 3v_t + 0.54$

$0.36 = 3v_t$

$v_t = 0.12$ m s^{-1}

12. $m = 0.06$ kg, $u = 18$ m s^{-1}, $v = -12$ m s^{-1}

(a) $\Delta p = m\Delta v = 0.06 \times (-12 - 18) = -1.8$ kg m s^{-1}

(b) $F = \dfrac{\Delta p}{\Delta t} = \dfrac{-1.8}{0.008} = -225$ N

13. $m_b = 0.03$ kg, $\mathrm{u}_b = 0$, $m_g = 3.5$ kg, $u_g = 0$, $v_b = 450$ m s^{-1}

$m_b u_b + m_g u_g = m_b v_b + m_g v_g$

$0 = (0.03 \times 450) + 3.5v_g$

$0 = 13.5 + 3.5v_g$

$v_g = -3.9 \text{ m s}^{-1}$

14. $m_r = m_b = m$, $u_r = 2 \text{ m s}^{-1}$, $u_b = -1 \text{ m s}^{-1}$, $v_r = 1.5 \text{ m s}^{-1}$ at 20°

Resolving momentum in the *x*-direction:

$mu_r + mu_b = mv_r + mv_b$

$2m - m = 1.5m \cos 20° + mv_b \cos \theta$

$m = 1.4m + mv_b \cos \theta$

$(1 - 1.4)m = mv_b \cos \theta$

$-0.4 = v_b \cos \theta$

Resolving in the *y*-direction:

$mu_r + mu_b = mv_r + mv_b$

$0 + 0 = m \times (1.5) \sin 20° + mv_b \sin \theta$

$0 = 0.51m + mv_b \sin \theta$

$-0.51 = v_b \sin \theta$

$\tan \theta = \frac{v_b \sin \theta}{v_b \cos \theta} = \frac{0.51}{0.4}$

$\theta = 51.9°$

$v_b^2 = (-0.4)^2 + (-0.51)^2$

$v_b = 0.65 \text{ m s}^{-1}$

Therefore the blue ball is deflected at an angle of 51.9° to its original line of motion at a speed of 0.65 m s^{-1}.

15. $m_b = 0.058$ kg, $u_b = 30 \text{ m s}^{-1}$, $m_r = 0.36$ kg, $u_r = 50 \text{ m s}^{-1}$, $v_r = 40 \text{ m s}^{-1}$

(a) $\Delta p = m\Delta v = 0.36\ (40 - 50) = -3.6$ Ns (loss)

(b) $\Delta p_b = -\Delta p_r = +3.6$ Ns (gain)

(c) $F = \frac{\Delta p}{\Delta t} = \frac{-3.6}{0.3} = -12$ N

(d) $\Delta v = \frac{\Delta p}{m} = \frac{-3.6}{0.058} = -62.1 \text{ m s}^{-1}$

$(v - u) = -62.1$

$v - 30 = -62.1$

$v = -32.1 \text{ m s}^{-1}$

16. (a) $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.004 \times 430^2 = 370$ J

(b) $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 7.25 \times 10^2 = 362$ J

(c) $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 1300 \times (22.2)^2 = 3.2 \times 10^5$ J

17. $m_p = 1.2$ kg, $m_b = 0.003$ kg, $u_b = u_p = 0$, $v_b = 420$ m s^{-1}

$0 = m_b v_b + m_g v_g$

$0 = (0.003 \times 420) + (1.2v_g)$

$0 = 1.26 + 1.2v_g$

$-1.26 = 1.2v_g$

$v_g = -1.05$ m s^{-1}

Therefore, after the firing, the gun's initial speed across the table is equal to 1.05 m s^{-1}.

$u = 1.05$ m s^{-1}, $m_g = 1.20$ kg, $v = 0$, $\mu = 0.12$

$F_f = \mu R$

$R = w = m_g g = 1.20 \times 9.8 = 11.76$ N

$F_f = 0.12 \times 11.76 = 1.41$ N

As friction is the only horizontally acting force on the gun,

$F_{net} = -F_f = -1.41$ N

$a = \frac{F}{m_g} = \frac{-1.41}{1.20} = -1.18$ m s^{-2}

$v^2 = u^2 + 2as$

$0 = (1.05)^2 + 2 \times (-1.18)$ s

$0 = 1.1 - 2.36s$

$2.36s = 1.1$

$s = 0.47$ m

The gun slides 0.47 m across the tabletop.

18. $m_A = m_B = m$, $u_A = 10$ m s^{-1}, $u_B = -5$ m s^{-1}

Considering the conservation of momentum:

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$(m \times 10) + (m \times -5) = m_A v_A + m_B v_B$

$10\,m - 5\,m = mv_A + mv_B$

$5 = v_A + v_B$

$v_A = 5 - v_B$ (1)

If the collision is elastic, then E_k will also be conserved:

$\frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$\frac{1}{2}m \times (10)^2 + \frac{1}{2}m \times (-5)^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$

$(10)^2 + (-5)^2 = v_A^2 + v_B^2$

$100 + 25 = v_A^2 + v_B^2$

$125 = v_A^2 + v_B^2$

Substitute (1) into this equation for v_A:

$125 = (5 - v_B)^2 + v_B^2$

$125 = 25 - 10v_B + 2v_B^2$

$100 = -10v_B + 2v_B^2$

$0 = v_B^2 - 5v_B - 50$

Solve for v_B by using the quadratic solution:

$$v_B = \frac{+5 \pm \sqrt{(-5)^2 - (4 \times 1 \times -50)}}{2 \times 1}$$

$v_B = +10\ \text{m s}^{-1}$ or $-5\ \text{m s}^{-1}$

If $v_B = -5\ \text{m s}^{-1}$, $v_A = 5 - (-5) = 10\ \text{m s}^{-1}$

In this case, it would be necessary for the two balls to continue to move through each other without any change in their motion. This is patently impossible, so we may discard this solution.

If $v_B = +10\ \text{m s}^{-1}$, $v_A = 5 - 10 = -5\ \text{m s}^{-1}$

This would mean that the balls have bounced off each other and are now moving in opposite directions than they were originally. Therefore, after collision, ball A is moving at 5 m s^{-1} and ball B is moving at 10 m s^{-1}.

19. $m_w = m_r = m$, $u_w = 4\ \text{m s}^{-1}$ at 0°, $u_r = 0$, $v_w = 2\ \text{m s}^{-1}$ at 340°, $\theta = 30°$

Resolving momentum in the x-direction:

$4m \cos 0° + 0 = 2m \cos 340° + mv_r \cos 30°$

$4m = 1.88m + 0.87mv_r$

$4 = 1.88 + 0.87v_r$

$2.12 = 0.87v_r$

$v_r = 2.44\ \text{m s}^{-1}$